# Drift instabilities in the pulses from cw mode-locked lasers

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(Received 7 October 1997)

The phenomenon of drift instability was recently identified in optics and hydrodynamics by several authors. Here we show that in optics also the temporal analogon exists. It can be found in all types of mode-locked lasers that may be described as two coupled clocks, i.e., actively mode locked, synchronously pumped, and certain passively (additive-pulse) mode-locked lasers. The relevant parameter is a timing mismatch as obtained, e.g., by cavity length misadjustment. Locking over a finite interval of mismatch is also observed, and turns out to be crucially important for the generation of stable pulses. Based on dedicated experiments and numerical simulations we explain the analogy, and the threshold behavior of locking. [S1063-651X(98)03307-8]

PACS number(s): 82.40.−q, 42.65.Sf, 42.60.Mi, 42.60.Fc

## INTRODUCTION

Spontaneous pattern formation, a hallmark of nonlinear systems, has been a subject of research since early in this century. For most of this time, investigation focused on fluid dynamics, but meanwhile it is well established that similar processes occur in other fields, including nonlinear optics.

In a typical situation, some variable that was influenced by a nonlinear medium acts back onto that medium. In the simplest case, the feedback acts on the same spatial position where the original interaction took place. In optics, that could imply an on-axis alignment of a feedback mirror. If, however, the feedback mechanism breaks the translational symmetry, i.e., acts back on a position slightly off the original one, typically a motion will occur of whatever structures or patterns are generated. A full description of this ''drift instability'' in optics was first given in [1], but see also [2] and [3]. It was then shown to occur in experiments involving nonlinear rubidium [4,5] or sodium vapor [6] and liquid-crystal light valves [7]. Similar phenomena occur in fluid dynamics [8].

An intuitive view would suggest that the patterns move with a velocity given by the feedback displacement divided by the feedback delay time. However, sometimes the situation is reported to be more complicated. Occasionally it was observed that for the drift velocity to be nonzero at all, a finite threshold displacement was required. In the case of [8] it was shown that the patterns were ''pinned'' to some imperfection in the apparatus, like scratches in a vessel wall or other deviations from spatial uniformity. In the optical case, dust specks in the beam path or diffraction effects at finite apertures [2] would be conceivable causes. One may consider such imperfections as not being a genuine part of the mechanism. However, when they are carefully eliminated, there is still a possibility that a nonzero drift velocity sets in only if some finite threshold displacement is exceeded due to some genuine physical effect [6,7]. For example it is shown in [6] that if the feedback tilt angle is less than some critical value, the drift velocity ''locks in'' at zero. A definite explanation of this locking phenomenon is missing so far.

In this paper we examine different types of mode-locked lasers and show that the temporal analogon of the drift instability can be observed, including the locking phenomenon. Here, however, an explanation of the locking mechanism can be given beyond the obvious observation that it is typical of nonlinear oscillators to synchronize over a finite interval of relative frequencies (i.e., inside an Arnold tongue). We also show that the technique known as ''coherent photon seeding'' presents the analogon to ''pinning.''

## I. MODE-LOCKED LASER SYSTEMS VIEWED AS COUPLED CLOCKS

Most techniques to generate mode locking rely on the interaction of two ''clocks.'' (On a different level, one can also think of all the coupled longitudinal modes of the laser as coupled clocks. This is not the present topic; we here adopt a time domain view of a mode-locked laser). In the actively mode-locked case, the laser resonator with its definite round-trip frequency is driven by an electronic circuit with its own independent frequency. In the synchronously pumped laser, similarly the laser resonator is driven by a stream of pump pulses with a certain repetition rate. And in passive mode-locking schemes like additive pulse mode locking that rely on two coupled cavities, both cavities have their individual round-trip frequency. In all three cases, the two clocks are coupled — either unidirectionally or bidirectionally. It is well known that for proper function the clock frequencies must be judiciously chosen and precisely maintained constant. On the other hand, it is remarkable that in an experimental situation one virtually never knows the precise resonator length detuning but relies on criteria like the resulting pulse shape for optimization.

The influence of deviations of the clock frequency setting from the optimal value has been a matter of much research (see, e.g., [9–14]). We will discuss the same point here and show that it can be seen in a different perspective that not only highlights the common properties of all these types of lasers, but provides an explanation for the locking mentioned above.

 

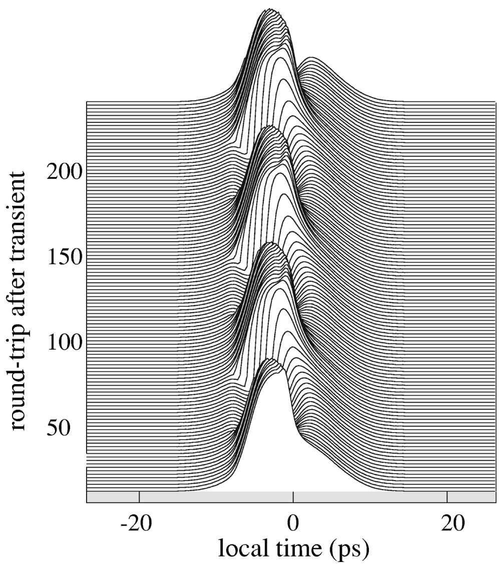


FIG. 1. Calculated pulse evolution in an APM laser in the drift regime. There is a period-2 (alternating pulse) behavior; for clarity, only every other round trip is shown. The figure shows how the pulses continuously drift to the left due to a cavity length mismatch; eventually they die out, and a new pulse grows on the right. This process repeats periodically. For details of the numerical model see [17].

## II. THE ADDITIVE-PULSE MODE-LOCKED LASER

Let us first consider additive pulse mode-locked (APM) lasers that have also been called soliton lasers, lasers with coupled-cavity mode locking, or with interferential mode locking (for an overview see, e.g., [15]). They consist of two coupled cavities, one of which contains the gain medium, the other a pulse-shaping nonlinearity. Most APM lasers are self-starting, i.e., the formation of ultrashort pulses is left entirely to self-organization. The pulse shaping mechanism relies on an interference of two replicas of the pulse, one from each cavity, which differ in their chirp. Stable trains of very short pulses have been routinely obtained from such lasers in several laboratories.

However, two coupled nonlinear oscillators have a potential to generate complex dynamical behavior. In fact, instabilities in the pulse trains of APM lasers were also observed and are reported in [16–19]. In our experiments we used a lamp-pumped Nd:YAG laser configured as an APM laser through the addition of a cavity containing a piece of single mode fiber. It turns out that much of the instabilities can be traced to effects resulting from cavity length mismatch, or in other words, clock detuning. For example, we showed that the quasiperiodic behavior first reported in [16] is produced by a slow oscillation of the pulse energies with a frequency proportional to the absolute value of the length mismatch [17]. The mechanism of these oscillations has a simple explanation: The pulse shapes are not entirely smooth but have certain perturbations in them. Due to the timing mismatch, these perturbations walk through the pulse from one round trip to the next. This modulates the total pulse energy, which is then seen to oscillate periodically. Figure 1 shows a simulation that illustrates this type of phenomenon.

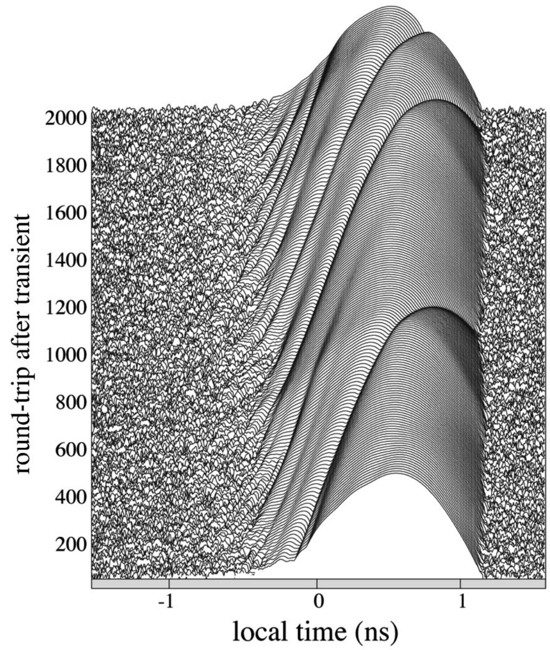


FIG. 2. Calculated pulse evolution in an actively mode-locked laser with a slightly too long cavity. The pulse evolution along 2000 round trips from the numerical model is plotted on a logarithmic scale. Perturbations walk across the pulse with constant velocity, but their starting position is random due to the spontaneous emission noise floor.

## III. THE ACTIVELY MODE-LOCKED LASER

Now we turn to actively mode-locked lasers in which an electronic driver (one clock) imposes its frequency on a laser cavity (the other clock) via a modulator. It is well known that again judicious choice of frequencies is required for good pulse generation.

Let us first present a few conclusions drawn from numerical simulations. In our algorithm, the pulse is modified on each round trip by saturable gain, bandwidth limitation as in [17], and the modulator; the modulator transmission curve is adapted from [20]. Parameter values are chosen to correspond to Nd:YAG. After transients have died out, the averaged autocorrelation function is calculated from the resulting pulse train.

The first obvious result is that the equilibrium position of the laser pulse shifts dramatically with respect to the time of maximum modulator transmission when the clock mismatch $\Delta T$ is changed only slightly. (Previous researchers have noticed such a shift, compare [21,22]). We find that the clock mismatch is magnified proportionally, by a factor of $\approx$600 (i.e., a mismatch of 1 ps causes a pulse delay of 600 ps.)

The second result is that the noise floor preceding the pulse seeds perturbations in the pulse; for a sufficient clock mismatch as in Fig. 2 these perturbations grow and walk, or drift, through the entire pulse with a constant velocity. A similar effect is well known for synchronously pumped lasers (see below), but we are not aware of any previous discussion for the actively mode-locked laser.

The third result is about this velocity. To quantify it, we simulated the average cross correlations between one pulse and the next and evaluated the position of the maximum,

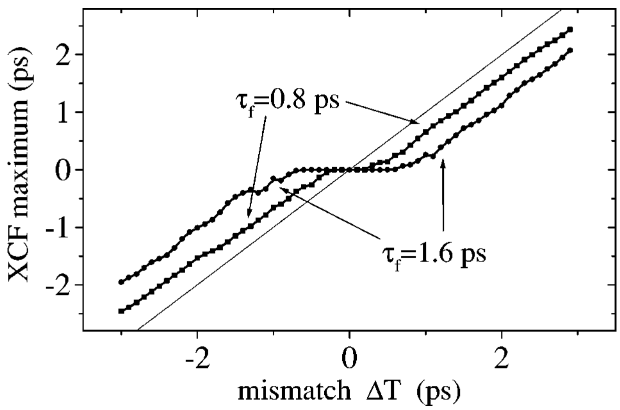


FIG. 3. Calculated temporal delay of the cross correlation peaks at different timing mismatches for an actively mode-locked laser with two different filter times. The timing mismatch can be obtained by tuning either the driver frequency or the resonator length.

which indicates the walk distance per round trip. This number is found to be equal to the mismatch amount, except for an offset (see Fig. 3). The offset is due to the locking of the drift velocity to zero over a finite interval, visible as the plateau in the center of the figure. The width of the locking regime turns out to be equal to the filter time $\tau_f$. $\tau_f$ is defined through the selectivity of the bandwidth limitation $\mathcal{S} = d^2T(\nu)/d\nu^2$, where $T(\nu)$ is the filter power transmission as a function of the optical frequency $\nu$, as $\tau_f = 1/(4\pi)\sqrt{\mathcal{S}}$ [23,24].

The fourth result is that not only the pulse envelope, but also its phase structure, get affected. Figure 4 presents an overview of the processes for clock mismatch less than (left column) or more than (right column) the threshold value of locking. The top row shows the pulse power envelope together with the transmission curve of the modulator. The second row shows the pulse power on a logarithmic scale, and the one at the bottom its phase. The shift of the pulse center from the modulator window maximum is clearly visible. It is also apparent that in the drift case each perturbation of the envelope corresponds to a jump in optical phase whereas in the locked case the pulse is symmetrical, Gauss-

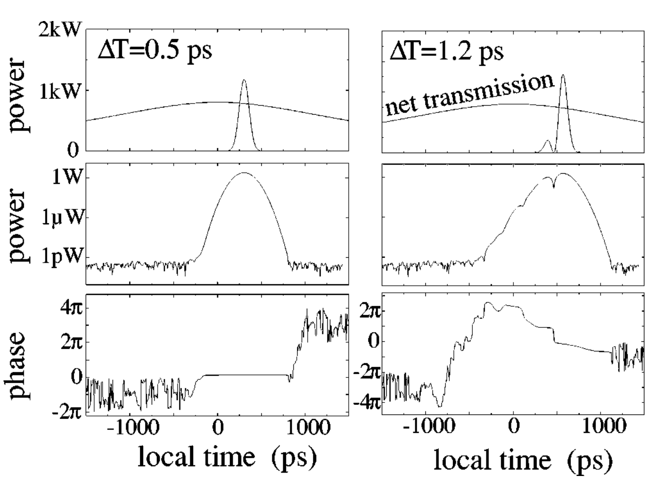


FIG. 4. Pulse profiles for an actively mode-locked laser in linear (top row) and logarithmic (center row) scale and the pulse phase (bottom row). Data are shown for two different timing mismatches: left, within; right, outside of locking range.

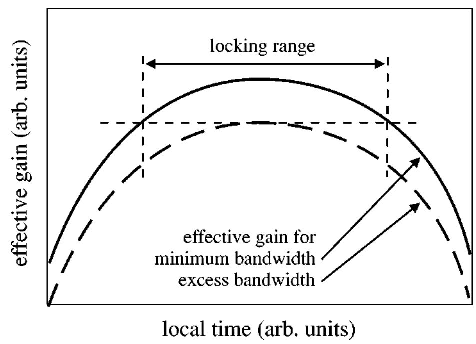


FIG. 5. Sketch of the effective gain as a function of time close to the moment of maximum modulator transmission. Fourier-limited pulses have higher effective gain than pulses with excess bandwidth, due to the cavity selectivity.

ian, and Fourier limited. Clearly, the optical bandwidth of the drifting pulse is larger than for the locked pulse.

For experimental verification we used a Quantronix model 416 lamp-pumped Nd:YAG laser. First, we measured the temporal shift between modulator drive maximum and pulse position as a function of cavity length detuning and find that the length detuning is magnified proportionally, by about 570 times, which is in almost perfect agreement with the simulation. Next, we checked the change of autocorrelation width and shape as the cavity length was varied (not shown); there is good agreement with simulation data. Since the pulsewidth was ≈100 ps and the possible pulling range of the cavity length detuning was limited to ≈2 ps, a meaningful experimental verification of a cross correlation delay expected to be of the same order turned out to be not feasible within the experimental accuracy.

We combine all these findings into the following explanation of the observed drifting and locking phenomenon: After startup, a clock mismatch leads to a small shift of the temporal pulse position with respect to the time of maximum modulator transmission at every round trip. In the case of a cavity longer (shorter) than required for exact match, the pulse is delayed (advanced) with respect to the modulator phase. After several round trips, this amounts to a considerable offset from the low-loss temporal window. Therefore, the pulse experiences a certain amount of extra attenuation. Moreover, the temporal slope of the modulator transmission is important: The extra attenuation is different across the pulse, so that it experiences a net restoring ''force'' that tends to push it back. The modulator gain gradient grows as the pulse moves out further; therefore, there can be an equilibrium position at which the pulse can settle down. It will acquire this position only after many round trips, hence the large magnification factor discussed above. Once the pulse has settled down in such an equilibrium position, the drift velocity is zero, and locking is observed.

However, this equilibrium is not always reached. As the clock mismatch is increased, the equilibrium position would be further and further out; this entails a growing additional attenuation. On the other hand, one has to keep in mind that in the presence of a given cavity selectivity (or filter time) the loss is different for a minimum bandwidth (Fourier-limited) pulse and a pulse with phase jumps and excess bandwidth. This is sketched in Fig. 5. At some clock mismatch the gain for the excess bandwidth structure begins to

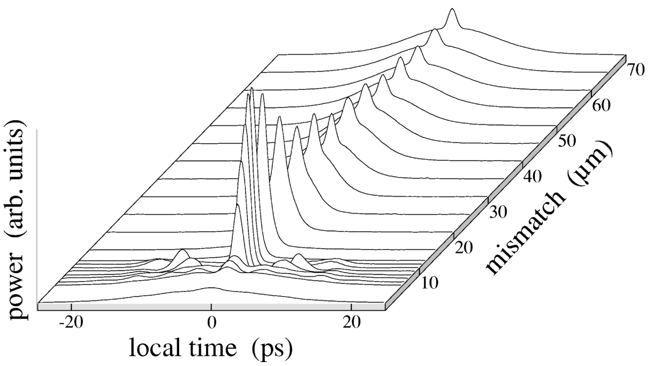


FIG. 6. Measured autocorrelation traces for a synchronously pumped laser at different length mismatches.

exceed the gain that a pulse would see at an equilibrium position. From that point on, there will be no stable pulse at equilibrium position. Instead, fluctuations grow to macroscopic size as they walk through the pulse. The threshold for drift is thus explained by the tendency of the laser to operate in whichever mode has the least loss.

## IV. THE SYNCHRONOUSLY PUMPED LASER

Finally we turn to the synchronously pumped laser in which a pump source and the pumped laser form a tandem. This arrangement was originally motivated by the need for tunable pulses when fixed frequency actively mode-locked lasers were available. Synchronously pumped laser systems have become a standard tool for spectroscopy etc., in many laboratories worldwide. From our present point of view we describe them as one clock driving another; it is natural to expect that small frequency detunings can have a large influence. We will show that essentially the same phenomena regarding drift instability and locking can be found as in the other types of lasers.

We performed experiments on a NaCl:$(F_2^+)_H$ color center that is described in detail in [24] (see also [23,25]). The pump source was an additive-pulse mode-locked Nd:YAG laser with pulsewidths around 10 ps (see [15,26]). The use of short pump pulses is beneficial because it reduces the formation of satellite pulses.

Figure 6 shows autocorrelation traces, and Fig. 7 cross correlation traces measured on the color center laser as a function of cavity length mismatch. As usual, the absolute resonator length is not known, and our choice of the origin of the mismatch axis is explained below. In the foreground of Fig. 6 the autocorrelation consists of a very broad single pulse. As the cavity length is increased, satellite pulses show up, but if one continues increasing the cavity length, they become weaker while the main pulses becomes narrower. The point of optimum performance is reached where the satellites are sufficiently suppressed but the main pulse has not appreciably broadened again. If one proceeds to lengthen the cavity beyond this point, the autocorrelation develops into a pedestal of increasing width with a coherence spike on top which, however, has constant width. This structure is indicative of incomplete mode locking.

As expected, the cross correlation trace (see Fig. 7) is symmetric and looks approximately the same as the autocorrelation trace except in the regime of the coherence spike.

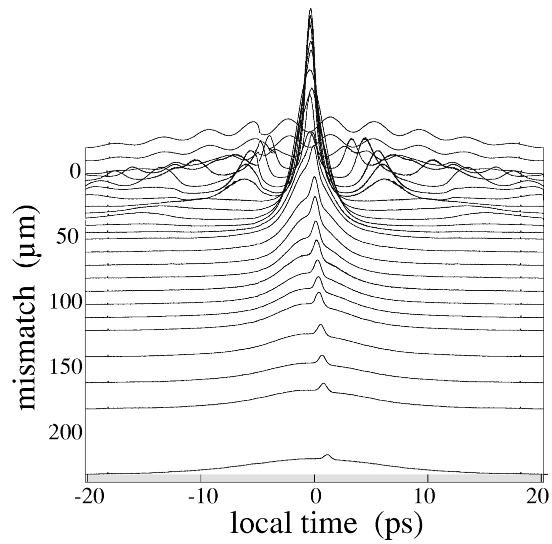


FIG. 7. Measured cross correlation functions (XCF) for a synchronously pumped laser at different length mismatches. One pulse is correlated with the next one. If the XCF equals the autocorrelation, the pulses are self-consistent; any asymmetry reveals pulse-to-pulse fluctuations. The delay of the spike is a measure of the drift speed of pulse substructure.

Note that the spike is off center in the cross correlation, indicative of some structure walking gradually across the pulse. This is the drift instability of the synchronously pumped laser.

From such cross correlation data we obtained the temporal delay of the spike as a function of the temporal mismatch; the result is shown in Fig. 8. The data fall on a straight line with a slope of unity that shows that the drift velocity is in fact equal to the timing mismatch, except for a possible offset. Whether there is such an offset one cannot tell due to the experimental uncertainty in the absolute position. Therefore, we performed numerical simulations for this situation too.

Our calculations are based on the approach of New [27–29], enhanced with an accurate implementation of the birefringent tuner plates [30]; further details are given in [24].

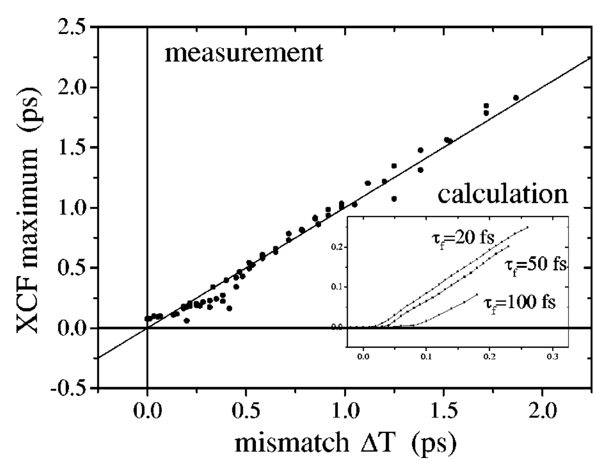


FIG. 8. Measured and calculated temporal delay of the cross correlation peaks at different length mismatches. One pulse is cross correlated with the next one. At zero delay the pulses are self-consistent; any other value reveals pulse-to-pulse fluctuations. The locking regime is not resolved in the experimental data.

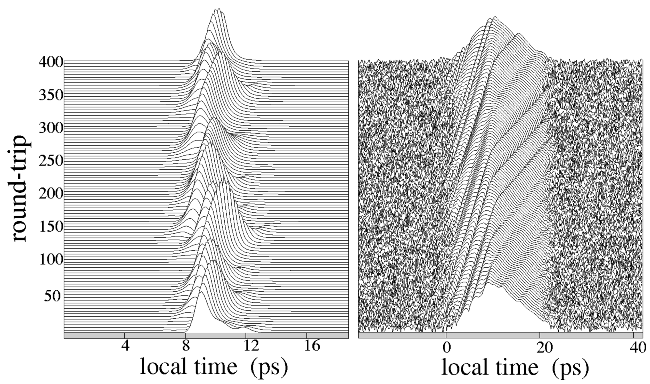


FIG. 9. Calculated pulse evolution in a slightly too long cavity. Linear and logarithmic scale. The origin of the time axis indicates the center of the pump pulse.

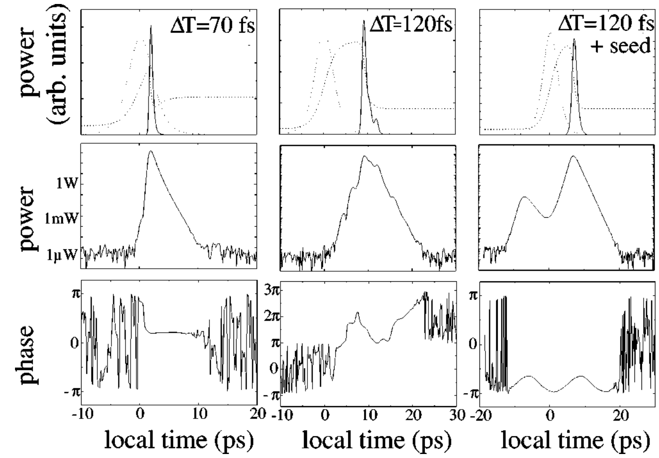


FIG. 10. Pulse profiles of a synchronously pumped laser (top row on a linear scale, center row, logarithmic), and the pulse phase (bottom row) for cavity length mismatch within (left column) and outside of (center column) locking regime, and with coherent photon seeding (right column). In the top row the pump pulse and the gain curve are displayed as dashed and dotted lines, respectively. Note that the pulse shape in the stable regime is close to a single-sided exponential as expected.

We first simulated the autocorrelation shapes as a function of mismatch; the result closely resembled the experimental data in Fig. 6. It also shows at which point the mismatch is zero; the origin of the mismatch axis in Figs. 6 and 7 was chosen accordingly. We then evaluated simulated cross correlation data for the drift velocity; the inset of Fig. 8 presents the result. Just as in the case of the actively mode-locked laser, there is a locking range up to a mismatch equal to the cavity filter time. The only difference is that in this case there are no data for negative mismatch; they would make little sense for a synchronously pumped laser.

Since the phenomena of locking and drift appear in this type of laser, too, it is warranted to look for the physical mechanism. From the data presented above and our numerical simulations we can safely draw the following conclusions: Consider a resonator length somewhat larger than for exact cold cavity match as in the regime of drift instabilities. The timing of the pulse results from the balance of two effects: the retardation due to the longer round-trip distance, and the acceleration due to the gain gradient. For the spontaneous emission noise there is only the retardation; thus noise is continuously injected into the leading edge of the pulse and continues shifting into the pulse center. Amplitude fluctuations might be damped out, but phase fluctuations are not. Thus, perturbations exist in the pulse that in time reach its center, and beyond. In several publications the importance of noise for the appearance of these so-called ''phase waves'' in SP lasers has been pointed out [31–33]. A simulation of this kind of behavior is shown in Fig. 9.

With this, the reason for drifting is well understood. Still, the cause of locking and its threshold behavior require an explanation. As in the case of the systems treated before, the effect of a gain gradient is responsible: The gain gradient across the laser pulse causes an enhanced effective pulse propagation velocity. Due to the gain gradient, the center of gravity of the pulse will be advanced or, in a manner of speaking, move ''faster than the speed of light'' [34]. Since the repetition rate of pulse generation is dictated by the pump repetition rate, it is the round-trip time including this extra push that must be adjusted to be equal to the repetition time. Therefore, the cold cavity needs to be slightly too long, by an amount found from the gain gradient across the pulse. (Several researchers have implied this fact; see, e.g., [9–12,22]). With increasing distance of the pulse from the pump pulse center the gradient increases. The timing mismatch continuously shifts the pulse away from the pump pulse, until the pushback due to the gradient balances the mismatch-induced shift. This equilibrium situation is illustrated in the left column of Fig. 10.

With increasing timing mismatch, there may be no equilibrium: before the gradient force suffices to push back the pulse position, the gain in the leading edge is enough to let the spontaneous emission noise grow macroscopic and walk into the pulse as perturbations (see middle column, Fig. 10). This will, at the same time, reduce the gain for the main pulse due to depletion of inversion. At some point the attenuation of the main pulse becomes larger than the extra loss of the amplified noise that is inflicted on it by the bandwidth limitation. This defines the threshold for drifting, and its dependence on cavity bandwidth.

We can now address the issue of pinning, or rather its temporal analogon. What is required is a condensation point *on the time axis*. To this end, one can superimpose a very small prepulse on the laser pulse that will swamp the fluctuations in the leading pulse edge with a coherent signal. This technique is known as ''coherent photon seeding'' (CPS) (see, e.g., [33,35–37]) and has been shown to reduce the noise in the pulse trains of synchronously pumped lasers considerably; it is illustrated in the third column in Fig 10.

## V. CONCLUSION

It may be intuitively clear that a timing mismatch causes perturbations to drift across the pulses. It is less obvious that a finite threshold mismatch exists below which this drift is locked into place. Moreover, even though a small timing mismatch from cold cavity resonance is required for optimum pulses, this optimum point seems to be within the locking range. If it were not for this fact, there would be no stable pulse trains at all from the lasers we discussed here.

Drift instability and locking have been discussed before in different context. Here we presented the temporal analogon and, in part inspired by the analogy, explained the locking phenomenon and its threshold behavior not only qualitatively but at least semiquantitatively. Full quantitative information, in view of the numerous parameters involved, is expected to be available only numerically.

**ACKNOWLEDGMENTS**

We thank T. Ackemann for helpful discussions, and R. Hartwigk for his participation in some of the experiments described here. Financial support from the Deutsche Forschungsgemeinschaft is gratefully acknowledged.

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